Regeneratively-constrained LQG control of passive networks ⋆

E. C. Warner *, **I. L. Cassidy** ** and **J. T. Scruggs** *

* *Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI, erwarner@umich.edu, jscruggs@umich.edu*
** *Lord Corporation, Cary, NC, Ian_Cassidy@lord.com*

Abstract: In this paper we consider the synthesis of optimal feedback controllers for a stochastically-excited passive electromechanical network, subject to the constraint that in stationarity, the feedback law must be realizable with a regenerative actuation system. Regenerative systems are similar to passive systems but their dynamic constraints are more relaxed, in the sense that they only need to conserve energy in the stationary average sense, rather than at every time instant. In this paper, we examine the design of optimal LQG controllers for passive networks controlled with regenerative actuation. We show that this problem may be posed as a multi-objective LMI problem. We also characterize how close a regeneratively-constrained optimal LQG feedback law is to a passive transfer function. The concepts are demonstrated on a simple example related to vibration suppression.

Keywords: Regeneration, passivity, vibration, optimal control

1. INTRODUCTION

In many control applications, the power and energy required for a given control design plays an important role in the assessment of its favorability. In applications where the energy available for control is stored locally (e.g., in a battery, supercapacitor, pressure accumulator, etc.) such issues become central to the viability of a control law. Such applications include many vibration suppression technologies, in which local energy storage is used to achieve energy-autonomy. This may be desirable from the point of view of reliability, such as for earthquake response control systems in civil structures, for which reliance on the external power grid introduces a significant vulnerability during seismic events. Energy-autonomy may also be desirable merely as a means of efficient system design, such as in automotive suspension control applications.

The price paid for energy-autonomy is that the domain of feasible control laws is constrained to include only those that do not exhaust their storage. To put this concept in more precise terms, consider the generic system diagram in Figure 1, illustrating a passive electromechanical network $\mathcal{N}$ excited exogenously through some vector $a(t) \in \mathbb{R}^{n_a}$ of dynamic inputs, and resulting in a vector $z(t) \in \mathbb{R}^{n_z}$ of performance quantities. We assume the system to be controlled through n_p ports, characterized by a vector $v(t) \in \mathbb{R}^{n_p}$ of potential variables, and a colocated vector $u(t) \in \mathbb{R}^{n_p}$ of flow variables. (For example, if $v(t)$ is a voltage vector then $u(t)$ is the vector of currents flowing into these voltages.) Vector $y(t) \in \mathbb{R}^{n_y}$ is comprised of the measurements available for feedback. We are then concerned with the design of a causal feedback law $K : y \to u$ that minimizes z under some measure.

The most straight-forward technique for designing an energy-autonomous feedback control law is to restrict the design domain to passive feedback laws, i.e.,

$$\mathcal{K}_p = \left\{ K : y \to u \,\middle|\, \int_0^T u^T(t) v(t) dt \leqslant 0, \right. \\ \left. \forall T \geqslant 0, y, v \in \mathcal{L}_2 \right\} \quad (1)$$

This is the domain of feedback laws which never inject cumulative energy into the network. The linear subdomain $\mathcal{K}_p^\ell \subset \mathcal{K}_p$ is comprised of all negative-real [1] colocated feedback laws; i.e.,

$$\mathcal{K}_p^\ell = \left\{ K : v \to u \,\middle|\, \hat{K}(s) + \hat{K}^H(s) \leqslant 0,\ \forall \operatorname{Re}(s) \geqslant 0 \right\} \quad (2)$$

It is a classical result (see, e.g., Darlington, 1999), that any such feedback law can be realized via a network of passive components. For example, in an electrical implementation, the feedback law could be implemented with ideal resistors, capacitors, inductors, transformers, and gyrators. At least on a theoretical level, a feedback law $K \in \mathcal{K}_p^\ell$ can therefore be implemented through classical network design, thus forgoing altogether the need for active control and an energy storage subsystem.

In the area of vibration control, such passive linear feedback implementations have been investigated in many contexts. Many mechanical techniques, such as tuned mass dampers, are ubiquitous in mechanical engineering

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[1] Note that, due to our sign convention for v and u, $K : v \to u$ constitutes positive feedback, and positive $u^T v$ denotes injection of power into the network. Thus K_p^ℓ is the domain of negative-real transfer functions. This is in contrast to the more usual convention of negative feedback between u and v which leads to the characterization of K_p^ℓ as the domain of all positive-real transfer functions.