Multi-objective structural control for simultaneous response suppression and power generation

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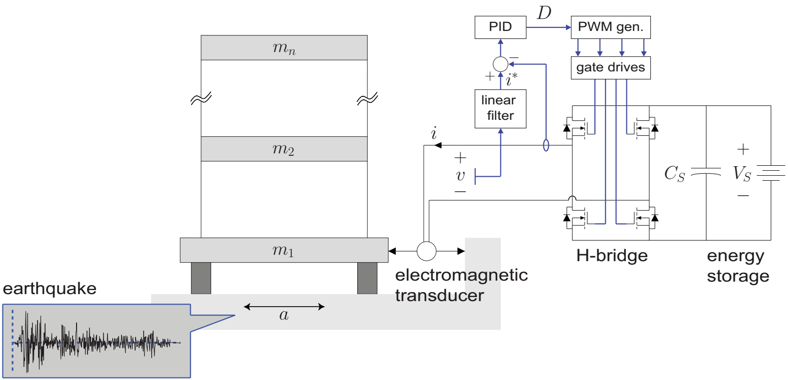
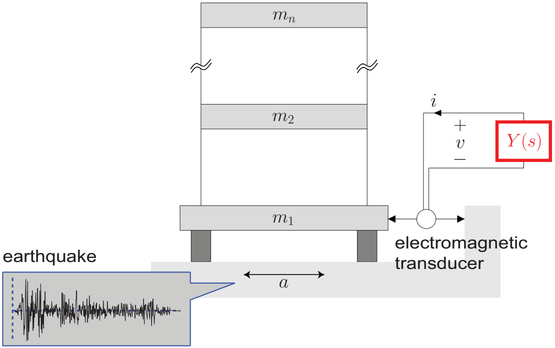
Active Collocated Structural Control



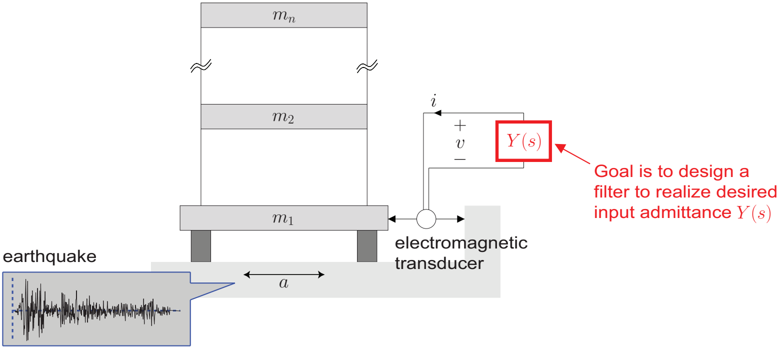
- Transducer consists of a ballscrew coupled to a permanent-magnet synchronous machine (Cassidy *et al.* (2011))
- Current controlled using PWM switching of four MOSFETs (bi-directional H-bridge drive)
- Linear feedback relates desired current i^* to measured voltage v

Active Collocated Structural Control



Goal is to design a filter to realize desired input admittance $Y(s)$

Active Collocated Structural Control



- Desired input admittance $Y(s)$ must be linear, but not necessarily passive
- Want to design $Y(s)$ to simultaneously suppress structural response quantities and generate power
- Can be framed using multi-objective optimization

Previous Work

Design of passive, semi-active, and regenerative controllers for vibration response suppression has been the subject of considerable research since the 1980s.

- Clipped-optimal control is the most popular semi-active control technique – first proposed for semi-active vehicle suspensions by Margolis (1982) and later by Karnopp (1983).
- Clipped-optimal control was investigated as a potential control technique for MR dampers by Dyke *et al.* (1996) and later by many, many others.
- Scruggs and Iwan (2005, 2006) developed regenerative damping controllers for networks of distributed actuators that can "share" power during seismic events.
- In Johnson and Erkus (2007), they enforced dissipative constraints for semi-active and active dampers using LMIs.

System Modeling

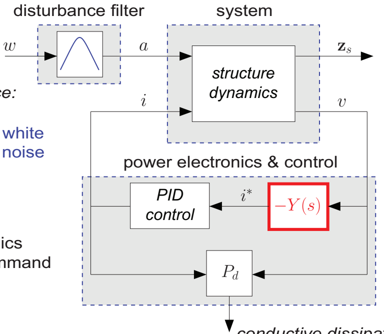
disturbance state space:

$$\dot{\mathbf{x}}_a = \mathbf{A}_a \mathbf{x}_a + \mathbf{B}_a w$$
$$a = \mathbf{C}_a \mathbf{x}_a$$

white noise

disturbance acceleration

note: power electronics can track current command such that $i^* \approx i$



conductive dissipation: $P_d = i^2 R$

structure state space:

$$\dot{\mathbf{x}}_s = \mathbf{A}_s \mathbf{x}_s + \mathbf{B}_s i + \mathbf{G}_s a$$
$$v = \mathbf{B}_s^T \mathbf{x}_s$$
$$\mathbf{z}_s = \mathbf{C}_s \mathbf{x}_s + \mathbf{D}_s i$$

transducer current

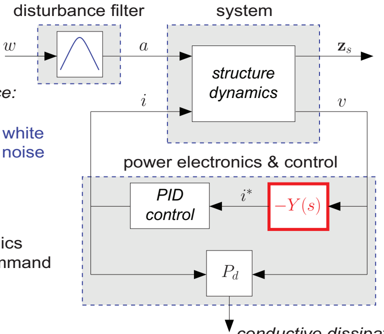
admittance state space:

$$\dot{\mathbf{x}}_Y = \mathbf{A}_Y \mathbf{x}_Y + \mathbf{B}_Y v$$
$$i^* = \mathbf{C}_Y \mathbf{x}_Y$$

command current

transducer voltage

System Modeling



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transducer current

admittance state space:

$$\mathbf{x}_Y = \mathbf{A}_Y \mathbf{x}_Y + \mathbf{B}_Y v$$
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command current

transducer voltage

Augmented structure and disturbance state space $\mathbf{x} = \begin{bmatrix} \mathbf{x}_s^T & \mathbf{x}_a^T \end{bmatrix}^T$ obeys

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}i + \mathbf{G}w$$
$$v = \mathbf{B}^T \mathbf{x}$$
$$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}i$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_s & \mathbf{G}_s \mathbf{C}_a \\ \mathbf{0} & \mathbf{A}_a \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_s \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_a \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \end{bmatrix},$$
$$\mathbf{D} = \mathbf{D}_s$$

Quadratic Performance Measures

Augmented structure and disturbance state space $\mathbf{x} = \begin{bmatrix} \mathbf{x}_s^T & \mathbf{x}_a^T \end{bmatrix}^T$ obeys

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}i + \mathbf{G}w \\ v &= \mathbf{B}^T\mathbf{x} \\ \mathbf{z} &= \mathbf{C}\mathbf{x} + \mathbf{D}i \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_s & \mathbf{G}_s\mathbf{C}_a \\ \mathbf{0} & \mathbf{A}_a \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_s \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_a \end{bmatrix},$$

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$$v = \mathbf{B}^T\mathbf{x} \qquad \mathbf{C} = \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \end{bmatrix},$$

$$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}i \qquad \mathbf{D} = \mathbf{D}_s$$

- $\mathbf{z}$ is a vector of dimensionless response quantities (e.g., interstory drifts and absolute accelerations)
- For structural response suppression, define performance measure as

$$J = \mathcal{E}\{\mathbf{z}^T\mathbf{z}\} = \mathcal{E}\left\{ \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix}^T \begin{bmatrix} \mathbf{C}^T\mathbf{C} & \mathbf{C}^T\mathbf{D} \\ \mathbf{D}^T\mathbf{C} & \mathbf{D}^T\mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix} \right\} = \mathcal{E}\left\{ \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix}^T \begin{bmatrix} \mathbf{Q}_z & \mathbf{S}_z \\ \mathbf{S}_z^T & R_z \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix} \right\}$$

- Can solve for the optimal LQG feedback controller $i^* = \mathbf{F}\mathbf{x}$ where

$$\mathbf{F} = -\frac{1}{R_z}\left(\mathbf{B}^T\mathbf{W} + \mathbf{S}_z^T\right)$$

and where $\mathbf{W} = \mathbf{W}^T > 0$ is the solution to the Riccati equation

$$\mathbf{A}^T\mathbf{W} + \mathbf{W}\mathbf{A} - \frac{1}{R_z}\left(\mathbf{W}\mathbf{B} + \mathbf{S}_z\right)\left(\mathbf{B}^T\mathbf{W} + \mathbf{S}_z^T\right) + \mathbf{Q}_z = \mathbf{0}$$

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$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}i + \mathbf{G}w \\ v &= \mathbf{B}^T\mathbf{x} \\ \mathbf{z} &= \mathbf{C}\mathbf{x} + \mathbf{D}i \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_s & \mathbf{G}_s\mathbf{C}_a \\ \mathbf{0} & \mathbf{A}_a \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_s \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_a \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \end{bmatrix},$$

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$$v = \mathbf{B}^T\mathbf{x} \qquad \mathbf{C} = \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \end{bmatrix},$$

$$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}i \qquad \mathbf{D} = \mathbf{D}_s$$

- For energy harvesting, define the average power generated $\bar{P}_{gen}$ as

$$\bar{P}_{gen} = -\mathcal{E}\{iv + i^2R\} = -\mathcal{E}\left\{ \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix}^T \begin{bmatrix} \mathbf{0} & \frac{1}{2}\mathbf{B} \\ \frac{1}{2}\mathbf{B}^T & R \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ i \end{bmatrix} \right\}$$

  where $-\mathcal{E}\{iv\}$ is the average transduction power and $-\mathcal{E}\{i^2R\}$ is the average conductive dissipation in the electronics
- Again, can solve for the optimal LQG feedback controller $i^* = \mathbf{K}\mathbf{x}$ where

$$\mathbf{K} = -\frac{1}{R}\mathbf{B}^T\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right)$$

  and where $\mathbf{P} = \mathbf{P}^T < 0$ is the solution to the non-standard Riccati equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \frac{1}{R}\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right)\mathbf{B}\mathbf{B}^T\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right) = \mathbf{0}$$

Multi-objective Optimization Problem

structure & disturbance state space:

$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}i + \mathbf{G}w$

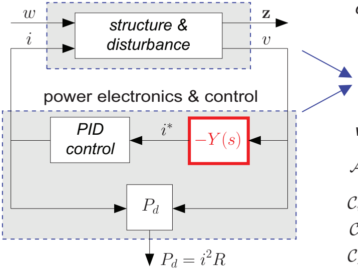
$v = \mathbf{B}^T\mathbf{x}$

$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}i$

admittance state space:

$\dot{\mathbf{x}}_Y = \mathbf{A}_Y\mathbf{x}_Y + \mathbf{B}_Y v$

$i^* = \mathbf{C}_Y\mathbf{x}_Y$



closed-loop state space:

$$\chi = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_Y \end{bmatrix} \qquad \begin{aligned} \dot{\chi} &= \mathcal{A}\chi + \mathcal{B}w \\ v &= \mathcal{C}_v\chi \\ i &= \mathcal{C}_i\chi \\ \mathbf{z} &= \mathcal{C}_z\chi \end{aligned}$$

where:

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{C}_Y \\ \mathbf{B}_Y\mathbf{B}^T & \mathbf{A}_Y \end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix}$$

$$\mathcal{C}_v = \begin{bmatrix} \mathbf{B}^T & \mathbf{0} \end{bmatrix}$$

$$\mathcal{C}_i = \begin{bmatrix} \mathbf{0} & \mathbf{C}_Y \end{bmatrix}$$

$$\mathcal{C}_z = \begin{bmatrix} \mathbf{C} & \mathbf{D}\mathbf{C}_Y \end{bmatrix}$$

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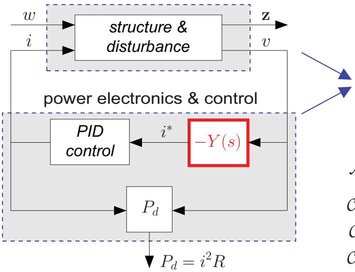

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In stationary stochastic response:

- Closed-loop system is linear → zero-mean Gaussian response
- System covariance matrix $\mathbf{S} = \mathcal{E}\{\chi\chi^T\}$ obeys $\mathcal{A}\mathbf{S} + \mathbf{S}\mathcal{A}^T + \mathcal{B}\mathcal{B}^T = \mathbf{0}$
- Can re-express objectives as

$$\text{Performance:}\quad J = \mathcal{E}\{\mathbf{z}^T\mathbf{z}\} = \text{tr}\{\mathcal{C}_z\mathbf{S}\mathcal{C}_z^T\}$$
$$\text{Power:}\quad \bar{P}_{gen} = -\mathcal{E}\{iv + i^2R\} = -\mathcal{C}_i\mathbf{S}\mathcal{C}_v^T - \mathcal{C}_i\mathbf{S}\mathcal{C}_i^T R$$

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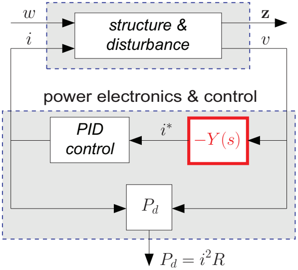

closed-loop state space:

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where:

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{C}_Y \\ \mathbf{B}_Y\mathbf{B}^T & \mathbf{A}_Y \end{bmatrix} \quad \mathcal{B} = \begin{bmatrix} \mathbf{G} \\ \mathbf{0} \end{bmatrix}$$

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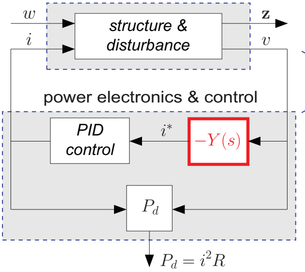

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where:

$$\mathcal{A} = \begin{bmatrix}\mathbf{A} & \mathbf{B}\mathbf{C}_Y\\ \mathbf{B}_Y\mathbf{B}^T & \mathbf{A}_Y\end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix}\mathbf{G}\\ \mathbf{0}\end{bmatrix}$$
$$\mathcal{C}_v = \begin{bmatrix}\mathbf{B}^T & \mathbf{0}\end{bmatrix}$$
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$$\mathcal{C}_z = \begin{bmatrix}\mathbf{C} & \mathbf{D}\mathbf{C}_Y\end{bmatrix}$$

Can express the multi-objective optimization problem as

$$\begin{aligned}
\text{Minimize :} \quad & \mathrm{tr}\{\mathcal{C}_z\mathbf{S}\mathcal{C}_z^T\}\\
\text{Domain :} \quad & \mathbf{A}_Y, \mathbf{B}_Y, \mathbf{C}_Y\\
\text{Constraints :} \quad & -\mathcal{C}_i\mathbf{S}\mathcal{C}_v^T - \mathcal{C}_i\mathbf{S}\mathcal{C}_i^T R > \bar{P}_{gen}^0\\
& \mathcal{A}\mathbf{S} + \mathbf{S}\mathcal{A}^T + \mathcal{B}\mathcal{B}^T = \mathbf{0}\\
& \max_i |\lambda_i(\mathcal{A})| \leqslant \omega_0
\end{aligned}$$

- $\bar{P}_{gen}^0$ is a specified amount of average power generated
- ω_0 is the operating bandwidth of the hardware

Optimization Method: LMI Approach

Can re-state the multi-objective optimization problem using LMIs as

Minimize : $\mathrm{tr}\{\mathcal{S}\}$

Domain : $\mathbf{A}_Y, \mathbf{B}_Y, \mathbf{C}_Y, \mathcal{P}, \mathcal{S}, \theta$

Constraints :

$$\begin{bmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B} \\ \mathcal{B}^T\mathcal{P} & -\mathbf{I} \end{bmatrix} < 0 \qquad \begin{bmatrix} \mathcal{S} & \mathcal{C}_z \\ \mathcal{C}_z^T & \mathcal{P} \end{bmatrix} > 0$$

$$\begin{bmatrix} \theta & \mathcal{C}_i - \mathcal{K} \\ \mathcal{C}_i^T - \mathcal{K}^T & \mathcal{P} \end{bmatrix} > 0 \qquad \begin{bmatrix} \omega_0\mathcal{P} & \mathcal{A}^T\mathcal{P} \\ \mathcal{P}\mathcal{A} & \omega_0\mathcal{P} \end{bmatrix} > 0$$

$$R\theta < \bar{P}_{gen}^{max} - \bar{P}_{gen}^{0}$$

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$$R\theta < \bar{P}_{gen}^{max} - \bar{P}_{gen}^{0}$$

- Recall that $\bar{P}_{gen}^{max} = -\mathbf{G}^T\mathbf{P}\mathbf{G}$ where $\mathbf{P} = \mathbf{P}^T < 0$ is the solution to

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \frac{1}{R}\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right)\mathbf{B}\mathbf{B}^T\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right) = \mathbf{0}$$

- and $\mathcal{K} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \end{bmatrix}$ where

$$\mathbf{K} = -\frac{1}{R}\mathbf{B}^T\left(\mathbf{P} + \tfrac{1}{2}\mathbf{I}\right)$$

Optimization Method: LMI Approach

Can re-state the multi-objective optimization problem using LMIs as

$$
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&\text{Minimize :} && \mathrm{tr}\{\mathcal{S}\} \\
&\text{Domain :} && \mathbf{A}_Y, \mathbf{B}_Y, \mathbf{C}_Y, \mathcal{P}, \mathcal{S}, \theta \\
&\text{Constraints :} && \begin{bmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B} \\ \mathcal{B}^T\mathcal{P} & -\mathbf{I} \end{bmatrix} < 0 \qquad \begin{bmatrix} \mathcal{S} & \mathcal{C}_z \\ \mathcal{C}_z^T & \mathcal{P} \end{bmatrix} > 0 \\
& && \begin{bmatrix} \theta & \mathcal{C}_i - \mathcal{K} \\ \mathcal{C}_i^T - \mathcal{K}^T & \mathcal{P} \end{bmatrix} > 0 \qquad \begin{bmatrix} \omega_0\mathcal{P} & \mathcal{A}^T\mathcal{P} \\ \mathcal{P}\mathcal{A} & \omega_0\mathcal{P} \end{bmatrix} > 0 \\
& && R\theta < \bar{P}_{gen}^{max} - \bar{P}_{gen}^{0}
\end{aligned}
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Optimization Method: LMI Approach

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& R\theta < \bar{P}_{gen}^{max} - \bar{P}_{gen}^{0}
\end{aligned}
$$

Notes:

1. First LMI enforces stationarity of the closed-loop system
2. Second LMI computes the stationary variances of $\mathbf{z}$
3. Third and fifth LMIs enforce that at least $\bar{P}_{gen}^{0}$ is harvested by the system
4. Fourth LMI restricts the operating bandwidth of the controller below ω_0

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&\text{Domain :} && \mathbf{A}_Y, \mathbf{B}_Y, \mathbf{C}_Y, \mathcal{P}, \mathcal{S}, \theta \\
&\text{Constraints :} && \begin{bmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B} \\ \mathcal{B}^T\mathcal{P} & -\mathbf{I} \end{bmatrix} < 0 \qquad \begin{bmatrix} \mathcal{S} & \mathcal{C}_z \\ \mathcal{C}_z^T & \mathcal{P} \end{bmatrix} > 0 \\
& && \begin{bmatrix} \theta & \mathcal{C}_i - \mathcal{K} \\ \mathcal{C}_i^T - \mathcal{K}^T & \mathcal{P} \end{bmatrix} > 0 \qquad \begin{bmatrix} \omega_0\mathcal{P} & \mathcal{A}^T\mathcal{P} \\ \mathcal{P}\mathcal{A} & \omega_0\mathcal{P} \end{bmatrix} > 0 \\
& && R\theta < \bar{P}_{gen}^{max} - \bar{P}_{gen}^{0}
\end{aligned}$$

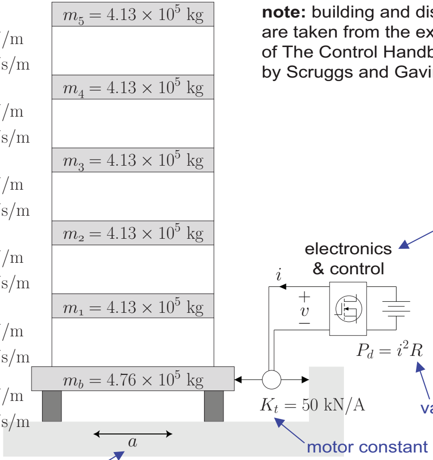
Notes:

1. First LMI enforces stationarity of the closed-loop system
2. Second LMI computes the stationary variances of $\mathbf{z}$
3. Third and fifth LMIs enforce that at least $\bar{P}_{gen}^{0}$ is harvested by the system
4. Fourth LMI restricts the operating bandwidth of the controller below ω_0

Through a change of variables (see Scherer et al. (1997)), problem can be made convex and solvable using standard LMI methods.

Example - Base Isolated Building

note: building and disturbance parameters are taken from the example in Chapter 30 of The Control Handbook, 2nd ed. (2010) by Scruggs and Gavin



operating bandwidth:

$$\omega_0 = \frac{2\pi}{10} f_s \approx 6.3 \times 10^3 \text{ rad/s}$$

disturbance state space:

$$\mathbf{A}_a = \begin{bmatrix} 0 & 1 \\ -\omega_a^2 & -2\zeta_a\omega_a \end{bmatrix}, \quad \mathbf{B}_a = \begin{bmatrix} 0 \\ \alpha \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} 0 & 2\zeta_a\omega_a \end{bmatrix}$$

where:

$\omega_a = 6.91$rad/s, $\zeta_a = 1.1$, $\alpha = 87.7$cm/s^2

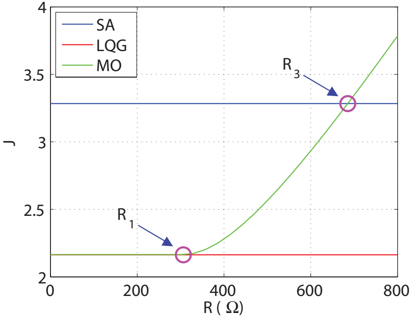
performance quantities:

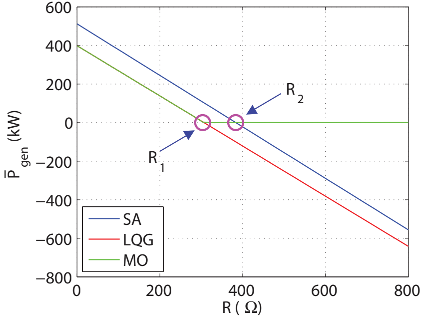
$$\mathbf{z} = \begin{bmatrix} d_b/\bar{d} & d_1/\bar{d} & a_b/\bar{a} & a_5/\bar{a} \end{bmatrix}$$

Results: $\bar{P}^0_{gen} = 0$

Case # 1:

- Mixed performance weights: $\{\bar{d}, \bar{a}\} = \{10\text{cm}, 0.1\text{g}\}$
- Fix $\bar{P}^0_{gen} = 0$ (i.e., "regenerative" constraint)
- Note: larger values of R correspond to less efficient electronics



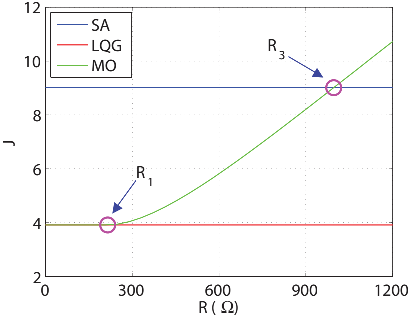


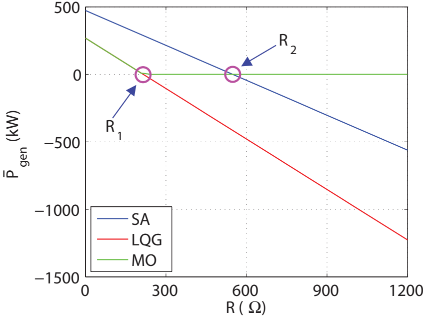
- $\{R_1, R_2, R_3\} = \{306\Omega, 383\Omega, 685\Omega\}$

Results: $\bar{P}^0_{gen} = 0$ (Continued)

Case # 2:

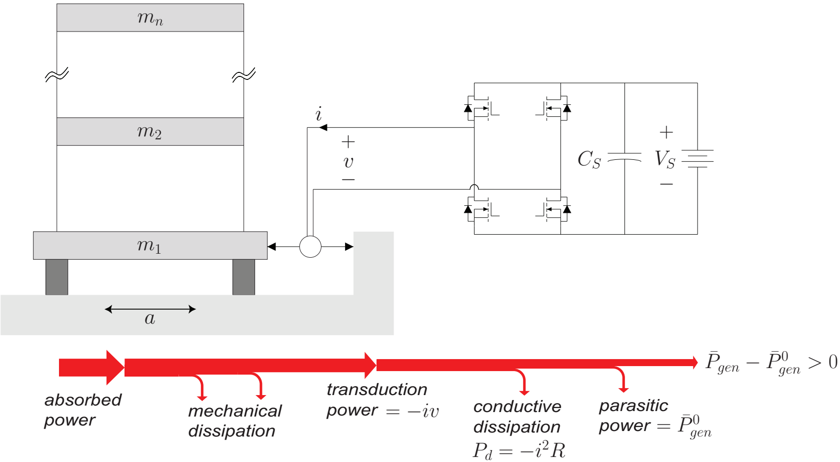
- Higher weight on accelerations: $\{\bar{d}, \bar{a}\} = \{10\text{cm}, 0.05\text{g}\}$
- Fix $\bar{P}^0_{gen} = 0$ (i.e., "regenerative" constraint)
- Note: larger values of R correspond to less efficient electronics





- $\{R_1, R_2, R_3\} = \{216\Omega, 549\Omega, 997\Omega\}$

Power Flow Diagram

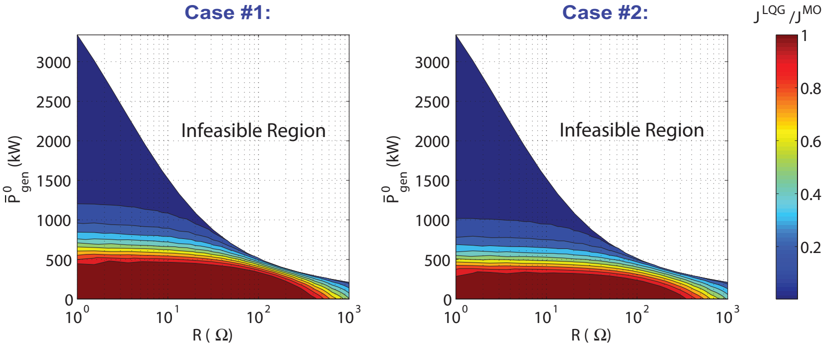


In some cases, may want $\bar{P}^0_{gen} > 0$ because some additional parasitic power is required to operate the power electronics and control intelligence.

Question: How big can we make $\bar{P}^0_{gen}$ before it starts to adversely affect J?

Results: $\bar{P}^0_{gen} > 0$

- For a given R, vary $\bar{P}^0_{gen} \in [0, \bar{P}^{max}_{gen}]$
- If $\bar{P}^0_{gen} > \bar{P}^{max}_{gen} \rightarrow$ problem is infeasible!



Conclusions and Future Work

Conclusions:

- Developed an LMI-based approach to simultaneous structural response suppression and power generation.
- For R values which result in LQG and SA controllers requiring power, can sacrifice some performance to ensure that $\bar{P}_{gen}^{MO} = 0$.
- Can set $\bar{P}_{gen}^{0} > 0$ to generate additional power to operate power electronics and control intelligence.

Future Work:

- Further investigate the effects of different weighting scenarios on the performance measures.
- Extend the theory to include non-quadratic loss models of the electronics (i.e., P_d is nonlinear).

Questions?